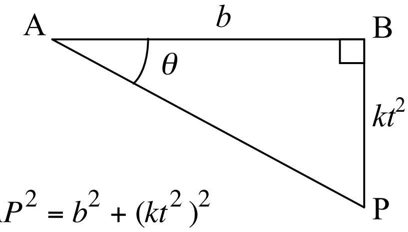
ASSESSMENT SCHEDULE (SAMPLE)

Mathematics with Calculus: Differentiate and use derivatives to solve problems (90635)

Evidence Statement

	Achievement Criteria	No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT	Differentiate functions and use differentiation to solve problems	One	$\frac{dy}{dx} = \frac{2x}{x^2 - 3}$		Or equivalent.	**ACHIEVEMENT:** Two of Code A
			At (2, 0), $\frac{dy}{dx} = 4$	A	No alternative.	
		Two	$f'(x) = 6x^2 - 42x + 72$ $f'(x) = 0$ for stat pts: $6x^2 - 42x + 72 = 0$ $x = 4$ or 3		First derivative found.	
			y coordinates of TPs: $f(3) = 76$ $f(4) = 75$		Coordinates of turning points found.	
			$f''(x) = 12x - 42$ $f''(3) = 36 - 42 = -6$ ie max $f''(4) = 48 - 42 = 6$ ie min		Nature of both turning points identified. Accept any method (incl CAO).	
			2 turning points: Maximum: (3, 76) Minimum: (4, 75)			
		Three	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $= 3\pi r$	A	$\frac{dA}{dt}$ determined in terms of r and evaluated when $r = 6$	
			When $r = 6$ $\frac{dA}{dt} = 18\pi \approx 56.5$ cm^2 per min	A	Or equivalent. Units not required.	

	Achievement Criteria	No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT WITH MERIT	Demonstrate knowledge, concepts and techniques of differentiation. Solve differentiation problems.	Four	$y = x^{\frac{1}{3}} e^{2x}$ $\frac{dy}{dx} = x^{\frac{1}{3}}.2e^{2x} + e^{2x}\frac{1}{3}x^{\frac{-2}{3}}$	A M1	Or equivalent.	**MERIT:** Achievement **plus** Three of Code M **or** All four Code M
		Five	(a) $\lim_{x \to 2} f(x) = 3$ (b) $f'(x) = 0$ when $-2 < x < 2$ and $x > 4$ (c) $f(x)$ not differentiable when $x = -2, 2, 3$ and 4	A M1	Accept two of 5a, 5b and 5c as sufficiency for Question five.	
		Six	$V = \frac{1}{3}\pi r^2 h, \quad h + r = 6$ $V = \frac{1}{3}\pi r^2(6 - r) \quad h = 6 - r$ $V = 2\pi r^2 - \frac{1}{3}\pi r^3$ $\frac{dV}{dr} = 4\pi r - \pi r^2$ $\frac{dV}{dr} = 0$ for max. $4\pi r - \pi r^2 = 0$ $r = 0$ or $r = 4$ Max Volume: $V = 32\pi - \frac{64}{3}\pi$ $= \frac{32}{3}\pi \approx 33.5 \text{ cm}^3$	A M2	First derivative found. Max volume evaluated. Units not required.	
		Seven	$A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3$ $\frac{dA}{dr} = 8\pi r \quad \frac{dV}{dr} = 4\pi r^2$ $\frac{dA}{dt} = \frac{dA}{dr}.\frac{dr}{dV}.\frac{dV}{dt}$ $= 8\pi r \times \frac{1}{4\pi r^2} \times 0.6$ $= \frac{1.2}{r}$ When $r = 2$, $\frac{dA}{dt} = 0.6 \text{ m}^2 \text{ sec}^{-1}$	A M2	Need to write an expression for $\frac{dA}{dt}$ and evaluate it for $r = 2$. Units not required.	

	Achievement Criteria	No.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT WITH EXCELLENCE	Solve problem(s) involving a combination of differentiation techniques.	Eight (a)	 $AP^2 = b^2 + (kt^2)^2$ $\tan\theta = \frac{kt^2}{b} \qquad \cos\theta = \frac{b}{\sqrt{b^2 + k^2t^4}}$ Differentiating implicitly $\sec^2\theta \times \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{k}{b} \times 2t$ $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{2kt}{b} \times \cos^2\theta$ $= \frac{2kt}{b} \times \frac{b^2}{b^2 + k^2t^4}$ $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{2bkt}{b^2 + k^2t^4}$ as required.	A M E	A suitable substitution is used to link the expressions: $\tan\theta = \frac{kt^2}{b}$ and $\cos\theta = \frac{b}{\sqrt{b^2 + k^2t^4}}$	**EXCELLENCE:** Merit **plus** Both Code E
		Eight (b)	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{2bkt}{b^2 + k^2t^4}$ $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{(b^2 + k^2t^4) \times 2bk - 2bkt \times 4k^2t^3}{(b^2 + k^2t^4)^2}$ $= \frac{2b^3k - 6bk^3t^4}{(b^2 + k^2t^4)^2}$ $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ is a max when $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = 0$ $2bk(b^2 - 3k^2t^4) = 0$ $b^2 - 3k^2t^4 = 0$ $t = \sqrt[4]{\frac{b^2}{3k^2}}$ $\theta = \tan^{-1}\left(\frac{kt^2}{b}\right)$ $\theta = \tan^{-1}\frac{k\left(\sqrt[4]{\frac{b^2}{3k^2}}\right)^2}{b}$ $\theta = \tan^{-1}\left(\frac{kb}{\sqrt{3}k} \times \frac{1}{b}\right)$ $\theta = \tan^{-1}\frac{1}{\sqrt{3}}$ $\theta = \frac{\pi}{6}$ as required.	A M E	The second derivative is found using the quotient rule. The resulting equation is solved for *t*. θ is found using the resulting *t* value. Allow minor errors.	